

Electromagnetic Corrections to the $\eta \rightarrow 3\pi$ Neutral Decay

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Abstract

Sutherland's theorem dictates that the contribution of the electromagnetic interaction to the decay process $\eta \rightarrow 3\pi^0$ is neglected with respect to the one coming from the difference between the up and down quark masses. In the framework of chiral perturbation theory including virtual photons, we calculated the main diagram concerning the exchange of a virtual photon between two intermediate charged pions. The correction induced by this diagram on the slope parameter amounts to 17% of the correction induced by the pure strong interaction at one-loop level. If this result is maintained when considering all the diagrams at the chiral order we are working, we can say without any doubt that Sutherland's theorem is strongly violated. As a direct consequence, any determination of light quark masses from the present decay *should* take into account the electromagnetic interaction.

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I. INTRODUCTION

The decay $\eta \rightarrow \pi^0 \pi^0 \pi^0$ is forbidden by isospin symmetry of the strong interaction. This symmetry is explicitly broken by electromagnetism on the one hand, and by the difference between up and down quark masses on the other hand. The decay amplitude takes then contributions from the electromagnetic fine structure constant α_{fs} and $m_d - m_u$. Sutherland's theorem states that the electromagnetic contribution is suppressed with respect to the other one [1, 2]. As a consequence, the decay $\eta \rightarrow 3\pi$ has been considered over the last thirty years as the golden process for the determination of the difference $m_d - m_u$ by comparing the theoretical predictions with the experimental measurements (See [3] and references therein). However, the data for the slope in the corresponding Dalitz plot show a big discrepancy between the experimental value and the theoretical predictions based on chiral perturbation theory [4]. This fact motivated the scientific community to perform very accurate measurements at the experimental level [5–7]. On the other side, several attempts have been conducted at the theoretical level. For instance, a model-independent study resting only on chiral symmetry and experimental data was performed in [8]. The authors of [3] used another approach based on data and analytic dispersive representation including first-order isospin breaking effects. Rescattering effects were considered in [9] through a modified non relativistic effective field theory beyond one loop including isospin-breaking corrections. It was claimed in the preceding citation that “the effect of photon exchange inside the charged pion loops on the Dalitz plot expansion is small, even on the scale of the other small isospin-breaking effects”. We believe, and will partially prove, that this is not the case. Our statement rests on the observation that the re-scattering effects between two pions in the final state is dominated by the Coulomb interaction which, in two different (but related) processes, gives a relatively valuable correction[10–12]. Motivated by this observation, we calculate in the present work the correction induced on the slope parameter of the decay $\eta \rightarrow 3\pi^0$ by photon exchange between two intermediate charged pions. To do so, the corresponding Feynman integral has been reduced to a set of five Master Integrals following Tarasov's reduction algorithm [13, 14], which is implemented in computer algebraic systems as a Mathematica package called Tarcerc [15]. The Master Integrals are then obtained in dimensional regularization by solving hypergeometric differential equations as reviewed in [16]. As a next step, we expand the decay amplitude around its value at the center of the

Dalitz plot and derive an analytic expression for the slope parameter. The value of the slope is then deduced in four dimensions by expanding around $D = 4$. We finally comment on our finding.

II. THE DECAY AMPLITUDE

The dynamics of the process

$$\eta(p) \longrightarrow \pi^0(p_1) + \pi^0(p_2) + \pi^0(p_3) \quad (1)$$

is studied in terms of the Mandelstam variables

$$s = (p - p_1)^2, \quad t = (p - p_2)^2, \quad u = (p - p_3)^2, \quad (2)$$

subject to the constraint

$$s + t + u = M_\eta^2 + 3M_{\pi^0}^2 \equiv 3s_0. \quad (3)$$

The decay amplitude \mathcal{M} is defined as

$$\langle \pi^0 \pi^0 \pi^0 | \eta \rangle = i(2\pi)^4 \delta^4(p_1 + p_2 + p_3 - p) \mathcal{M}, \quad (4)$$

and can be written by symmetry considerations like

$$\mathcal{M}(s, t, u) = M(s) + M(t) + M(u). \quad (5)$$

In the framework of chiral perturbation theory, we write the s -channel amplitude as

$$M(s) = -\frac{B_0(m_d - m_u)}{3\sqrt{3}F_\pi^2} [1 + \delta_{\text{str}}(s) + \delta_{\text{em}}(s) + \delta_\gamma(s)] + \tilde{\delta}_{\text{em}}(s), \quad (6)$$

where δ_{str} is of $\mathcal{O}(p^2)$, δ_{em} of $\mathcal{O}(e^2)$, and $\tilde{\delta}_{\text{em}}$ of $\mathcal{O}(e^2 p^2)$ in the chiral counting and have been calculated in [17], [18], and [8], respectively. The correction δ_γ is of $\mathcal{O}(e^2 p^2)$ and is supposed to be small compared to the others. The main diagram contributing to δ_γ is sketched in Fig. 1. Working in the framework of chiral perturbation theory including photons [19], we find that, up to polynomials in the kinematic invariants, the main diagram is given in terms of D -dimensional integrals of the form

$$\frac{1}{\pi^D} \iint \frac{d^D k_1 d^D k_2 (k_1^2)^i (k_2^2)^j (q \cdot k_1)^k (q \cdot k_2)^l (k_1 \cdot k_2)^m}{(k_1^2 - M_\pi^2)(k_2^2 - M_\pi^2)[(k_1 - q)^2 - M_\pi^2][(k_2 - q)^2 - M_\pi^2](k_1 - k_2)^2}. \quad (7)$$

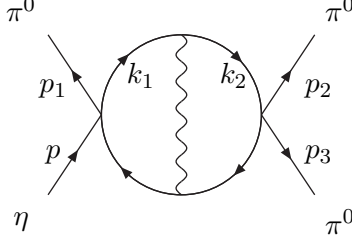


Figure 1: In this diagram $q \equiv p - p_1 = p_2 + p_3$.

Applying Tarasov's reduction algorithm, integrals (7) are reduced to a set of five Master Integrals, J_1 , J_2 , JT , J^2 , and T^2 with

$$T = \frac{1}{\pi^{D/2}} \int \frac{d^D k}{k^2 - M_\pi^2}, \quad (8)$$

$$J = \frac{1}{\pi^{D/2}} \int \frac{d^D k}{(k^2 - M_\pi^2)[(k - q)^2 - M_\pi^2]}, \quad (9)$$

$$J_1 = \frac{1}{\pi^D} \iint \frac{d^D k_1 d^D k_2}{(k_2^2 - M_\pi^2)[(k_1 - q)^2 - M_\pi^2](k_1 - k_2)^2}, \quad (10)$$

$$J_2 = \frac{1}{\pi^D} \iint \frac{d^D k_1 d^D k_2 (k_1 \cdot k_2)}{(k_2^2 - M_\pi^2)[(k_1 - q)^2 - M_\pi^2](k_1 - k_2)^2}. \quad (11)$$

We found that the contribution of the main diagram to δ_γ takes the following form in D dimensions

$$\delta_\gamma = -\frac{\alpha_{\text{fs}}}{3F_\pi^2} \frac{1}{M_{\pi^0}^2(M_\eta^2 - M_{\pi^0}^2)} \frac{1}{s^3 \sigma^4(s)} \times \frac{1}{(D-4)(D-3)} \frac{1}{(4\pi)^{D-1}} (c_1 J_1 + c_2 J_2 + c_3 JT + c_4 J^2 + c_5 T^2), \quad (12)$$

where

$$\sigma(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}, \quad (13)$$

and,

$$\begin{aligned} c_1 = & -(-384D^2 M_\pi^{10} + 1536DM_\pi^{10} - 1152M_\pi^{10} + 608D^2 s M_\pi^8 - 2528Ds M_\pi^8 \\ & + 864s M_\pi^8 + 184D^2 s^2 M_\pi^6 - 624Ds^2 M_\pi^6 + 2240s^2 M_\pi^6 - 650D^2 s^3 M_\pi^4 + 2810Ds^3 M_\pi^4 \\ & - 3408s^3 M_\pi^4 + 257D^2 s^4 M_\pi^2 - 1299Ds^4 M_\pi^2 + 1564s^4 M_\pi^2 + 12D^2 s^5 - 42Ds^5 + 48s^5), \\ c_2 = & 2(-576D^2 M_\pi^8 + 2880DM_\pi^8 - 3456M_\pi^8 + 1360D^2 s M_\pi^6 - 6880Ds M_\pi^6 \\ & + 7776s M_\pi^6 - 980D^2 s^2 M_\pi^4 + 5168Ds^2 M_\pi^4 - 5568s^2 M_\pi^4 + 187D^2 s^3 M_\pi^2 \\ & - 1213Ds^3 M_\pi^2 + 1356s^3 M_\pi^2 + 36D^2 s^4 - 126Ds^4 + 144s^4), \end{aligned}$$

$$\begin{aligned}
c_3 &= -s^2\sigma^2(s)(56D^2M_\pi^6 - 376DM_\pi^6 + 416M_\pi^6 - 66D^2sM_\pi^4 + 522DsM_\pi^4 - 600sM_\pi^4 \\
&\quad + D^2s^2M_\pi^2 - 119Ds^2M_\pi^2 + 148s^2M_\pi^2 + 12D^2s^3 - 42Ds^3 + 48s^3), \\
c_4 &= 3s^2\sigma^2(s)M_\pi^2(3s - 4M_\pi^2)(s - M_\pi^2)(D - 3)(-4DM_\pi^2 + 8M_\pi^2 + 3Ds - 8s), \\
c_5 &= -192D^2M_\pi^8 + 1152DM_\pi^8 - 1536M_\pi^8 + 560D^2sM_\pi^6 - 3312DsM_\pi^6 \\
&\quad + 3712sM_\pi^6 - 396D^2s^2M_\pi^4 + 2568Ds^2M_\pi^4 - 2640s^2M_\pi^4 + 31D^2s^3M_\pi^2 \\
&\quad - 489Ds^3M_\pi^2 + 524s^3M_\pi^2 + 36D^2s^4 - 126Ds^4 + 144s^4.
\end{aligned} \tag{14}$$

III. THE MASTER INTEGRALS

Three of the Master Integrals are given in terms of one-loop integrals T and J ,

$$T = -iM_\pi^{D-2}\Gamma(1 - D/2), \tag{15}$$

$$J = i \left\{ 1 - M_\pi^{D-4}\Gamma(1 - D/2) + \sigma \left[\ln \left(\frac{1 - \sigma}{1 + \sigma} \right) + i\pi \right] \right\}. \tag{16}$$

The remaining two integrals are found to satisfy the system of first order differential equations,

$$\frac{dJ_1}{ds} = \frac{(5 - 2D)s + 2(3 - 2D)M_\pi^2}{s(4M_\pi^2 - s)}J_1 + \frac{6(D - 2)}{s(4M_\pi^2 - s)}J_2 + \frac{2(D - 2)}{s(4M_\pi^2 - s)}T^2, \tag{17}$$

$$\begin{aligned}
\frac{dJ_2}{ds} &= -\frac{8(D - 1)M_\pi^4 + 2(3D - 8)sM_\pi^2 + (D - 2)s^2}{4s(4M_\pi^2 - s)}J_1 \\
&\quad + \frac{D - 2}{2s} \left(\frac{8M_\pi^2 + s}{4M_\pi^2 - s} \right) J_2 + \frac{D - 2}{4s} \left(\frac{4M_\pi^2 + s}{4M_\pi^2 - s} \right) T^2.
\end{aligned} \tag{18}$$

Such a system is equivalent to a second order differential equation

$$\begin{aligned}
\frac{d^2J_1}{ds^2} &= -\frac{(D - 2)^2}{2s^2(4M_\pi^2 - s)}T^2 - \frac{3(4M_\pi^2 + Ds - 4s)}{2s(4M_\pi^2 - s)}\frac{dJ_1}{ds} \\
&\quad + \frac{(D - 3)[2(D - 2)M_\pi^2 + (D - 4)s]}{2s^2(4M_\pi^2 - s)}J_1.
\end{aligned} \tag{19}$$

Differentiating once more, we obtain a third order differential equation

$$\begin{aligned}
\frac{d^3J_1}{ds^3} &= \frac{(D - 4)(D - 3)}{2s^2(4M_\pi^2 - s)}J_1 - \frac{28M_\pi^2 + 3Ds - 18s}{2s(4M_\pi^2 - s)}\frac{d^2J_1}{ds^2} \\
&\quad + \frac{2D^2M_\pi^2 + sD^2 - 10DM_\pi^2 - 13sD + 36s}{2s^2(4M_\pi^2 - s)}\frac{dJ_1}{ds}.
\end{aligned} \tag{20}$$

Making the change of variable,

$$x = -\frac{s}{4M_\pi^2}, \tag{21}$$

we find that J_1 satisfies the hypergeometric differential equation

$$x^2(1+x)\frac{d^3J_1}{dx^3} - x\left[\frac{3(D-6)}{2}x - \frac{7}{2}\right]\frac{d^2J_1}{dx^2} + \left[\frac{(D-4)(D-9)}{2}x - \frac{D(D-5)}{4}\right]\frac{dJ_1}{dx} + \frac{(3-D)(4-D)}{2}J_1 = 0. \quad (22)$$

The general solution of such an equation is

$$\begin{aligned} J_1 = & {}_3F_2 \left[\begin{matrix} (3-D), (4-D)/2, 1 \\ D/2, (5-D)/2 \end{matrix} ; -x \right] A \\ & + (-x)^{-(D-2)/2} {}_3F_2 \left[\begin{matrix} (8-3D)/2, (3-D), (4-D)/2 \\ (4-D)/2, (7-2D)/2 \end{matrix} ; -x \right] B \\ & + (-x)^{(D-3)/2} {}_3F_2 \left[\begin{matrix} (3-D), 1/2, (D-1)/2 \\ (2D-3)/2, (D-1)/2 \end{matrix} ; -x \right] C. \end{aligned} \quad (23)$$

In order to determine the integration constants A , B , and C , we notice that

$$J_1(0) = \frac{1}{2M_\pi^2} \left(\frac{D-2}{D-3} \right) T^2, \quad (24)$$

is regular for $2 < D < 4$. It follows that $B = C = 0$ since the terms $(-x)^{-(D-2)/2}$ and $(-x)^{(D-3)/2}$ are divergent for $x = 0$ and in the range in question. Moreover, ${}_3F_2$ is equal to one for $x = 0$. This means that $A = J_1(0)$ and

$$J_1 = \frac{1}{2M_\pi^2} \left(\frac{D-2}{D-3} \right) {}_3F_2 \left[\begin{matrix} (3-D), (4-D)/2, 1 \\ D/2, (5-D)/2 \end{matrix} ; -x \right] T^2. \quad (25)$$

Having J_1 , we obtain J_2 from

$$J_2 = -\frac{1}{3}T^2 + \frac{2M_\pi^2}{3(D-2)}x(1+x)\frac{dJ_1}{dx} - \frac{M_\pi^2[2(2D-5)x+3-2D]}{3(D-2)}J_1, \quad (26)$$

by the use of

$$\begin{aligned} \frac{d}{dx} {}_3F_2 \left[\begin{matrix} (3-D), (4-D)/2, 1 \\ D/2, (5-D)/2 \end{matrix} ; -x \right] = \\ \frac{2(D-3)(D-4)}{D(D-5)} {}_3F_2 \left[\begin{matrix} (4-D), (6-D)/2, 2 \\ (D+2)/2, (7-D)/2 \end{matrix} ; -x \right]. \end{aligned} \quad (27)$$

We get

$$J_2 = \left\{ -\frac{2(2D-5)x+3-2D}{6(D-3)} {}_3F_2 \left[\begin{matrix} (3-D), (4-D)/2, 1 \\ D/2, (5-D)/2 \end{matrix} ; -x \right] \right.$$

$$+\frac{2(D-4)}{3D(D-5)}x(x+1)_3F_2\left[\begin{matrix}(4-D), (6-D)/2, 2 \\ (D+2)/2, (7-D)/2\end{matrix}; -x\right]-\frac{1}{3}\Bigg\}T^2. \quad (28)$$

IV. THE SLOPE PARAMETER

The amplitude is expanded around its value at the center of the Dalitz plot as

$$|\mathcal{M}(s, t, u)|^2 = |\mathcal{M}(s_0, s_0, s_0)|^2 \{1 + 2\alpha z\}, \quad (29)$$

where

$$z = \frac{3}{2M_\eta^2(M_\eta - 3M_{\pi^0})^2} \{(s - s_0)^2 + (t - s_0)^2 + (u - s_0)^2\}. \quad (30)$$

We write the slope parameter as

$$\alpha = \alpha_{\text{str}} + \alpha_{\text{em}} + \alpha_\gamma + \frac{M_{\pi^\pm}^2 - M_{\pi^0}^2}{B_0(m_d - m_u)} \tilde{\alpha}_{\text{em}}. \quad (31)$$

The contribution of the main diagram to the slope parameter is

$$\alpha_\gamma = \frac{M_\eta^2}{9}(M_\eta - 3M_{\pi^0})^2 \text{Re } \delta_\gamma''(s_0). \quad (32)$$

We can express α_γ in terms of the five Master Integrals thanks to the differential equations satisfied by the latter. Up to now, we have considered only the differential equations satisfied by J_1 and J_2 . It is easy to check that the one-loop integral satisfies the following differential equation

$$\frac{dJ}{ds} = \frac{(D-4)s + 4M_\pi^2}{2s(s - 4M_\pi^2)}J - \frac{D-2}{s(s - 4M_\pi^2)}T. \quad (33)$$

Differentiating δ_γ twice with respect to s and using the preceding relation at each step, we obtain

$$\begin{aligned} \alpha_\gamma = & -\frac{\alpha_{\text{fs}}}{108F_\pi^2} \frac{M_\eta^2(M_\eta - 3M_{\pi^0})^2}{M_{\pi^0}^2(M_\eta^2 - M_{\pi^0}^2)} \frac{1}{s_0^7 \sigma^8(s_0)} \times \\ & \frac{1}{(D-4)(D-3)} \frac{1}{(4\pi)^{D-1}} \text{Re} (d_1 J_1 + d_2 J_2 + d_3 J T + d_4 J^2 + d_5 T^2)_{s=s_0}, \end{aligned} \quad (34)$$

with

$$\begin{aligned} d_1 = & 6144(D^4 - 16D^3 + 86D^2 - 176D + 105)M_\pi^{14} - 512(25D^4 - 360D^3 + 1511D^2 \\ & - 2214D + 621)sM_\pi^{12} + 128(48D^4 - 374D^3 + 257D^2 + 3240D - 5856)s^2M_\pi^{10} \\ & - 32(50D^4 + 2009D^3 - 17080D^2 + 43918D - 36964)s^3M_\pi^8 + 8(397D^4 - 684D^3 \end{aligned}$$

$$\begin{aligned}
& -15994D^2 + 65826D - 69384)s^4M_\pi^6 + 2(201D^4 + 6971D^3 - 60836D^2 \\
& + 161172D - 139248)s^5M_\pi^4 + (-1697D^4 + 16825D^3 - 62006D^2 \\
& + 101120D - 61664)s^6M_\pi^2 + 6D(-2D^3 + 11D^2 - 22D + 16)s^7, \\
d_2 = & 2[-9216(D^4 - 17D^3 + 101D^2 - 247D + 210)M_\pi^{12} + 256(103D^4 - 1527D^3 \\
& + 7880D^2 - 17199D + 13446)sM_\pi^{10} - 64(454D^4 - 5634D^3 + 25631D^2 \\
& - 50730D + 36744)s^2M_\pi^8 + 48(398D^4 - 3551D^3 + 12214D^2 - 19708D \\
& + 12600)s^3M_\pi^6 + 4(-2491D^4 + 18774D^3 - 43814D^2 + 26916D + 11088)s^4M_\pi^4 \\
& + (2971D^4 - 30543D^3 + 114482D^2 - 188208D + 115296)s^5M_\pi^2 \\
& + 18D(2D^3 - 11D^2 + 22D - 16)s^6], \\
d_3 = & s^2\sigma^2(s)[384(10D^3 - 77D^2 + 207D - 172)M_\pi^{10} + 32(18D^4 - 320D^3 \\
& + 1733D^2 - 3751D + 2764)sM_\pi^8 - 8(187D^4 - 1741D^3 + 5696D^2 \\
& - 7898D + 3816)s^2M_\pi^6 + 6(209D^4 - 1951D^3 + 5866D^2 - 6400D \\
& + 1568)s^3M_\pi^4 + (-325D^4 + 3953D^3 - 16398D^2 + 29008D \\
& - 18848)s^4M_\pi^2 + 6D(-2D^3 + 11D^2 - 22D + 16)s^5], \\
d_4 = & -12M_\pi^2s^2\sigma^2(s)[512(D^2 - 5D + 6)M_\pi^{10} + 128(D^3 - 11D^2 \\
& + 36D - 36)sM_\pi^8 + 16(D^4 - 16D^3 + 91D^2 - 216D + 180)s^2M_\pi^6 \\
& - 8(5D^4 - 38D^3 + 92D^2 - 65D - 12)s^3M_\pi^4 + (D - 3)^2(33D^2 \\
& - 68D - 64)s^4M_\pi^2 - 3(D - 3)^2(3D^2 - 14D + 16)s^5], \\
d_5 = & -3072(D^4 - 18D^3 + 115D^2 - 306D + 280)M_\pi^{12} + 256(37D^4 - 615D^3 + 3401D^2 \\
& - 7860D + 6444)sM_\pi^{10} - 64(183D^4 - 2517D^3 + 12185D^2 - 25152D + 18772)s^2M_\pi^8 \\
& + 16(568D^4 - 5294D^3 + 18567D^2 - 30394D + 19784)s^3M_\pi^6 + 4(-1386D^4 + 10909D^3 \\
& - 25796D^2 + 15140D + 7808)s^4M_\pi^4 + 5(359D^4 - 3883D^3 + 14986D^2 - 25136D \\
& + 15648)s^5M_\pi^2 + 18D(2D^3 - 11D^2 + 22D - 16)s^6. \tag{35}
\end{aligned}$$

V. RESULTS AND CONCLUSIONS

We first expand δ_γ around $D = 4$ and then use the following numerical values

$$\alpha_{\text{fs}} = \frac{1}{137.04}, \quad (F_\pi, M_{\pi^0} \equiv M_\pi, M_\eta) = (92.42, 139.57, 547.30) \text{ MeV}. \tag{36}$$

We found that the amplitude, δ_γ , possesses a pole at the lower bound of the allowed kinematical region, that is, for $s = 4M_{\pi^0}^2$. At the upper bound, $s = (M_\eta - M_{\pi^0})^2$, the calculated electromagnetic correction amounts to 3-4% of the Tree level amplitude. This points out that the present correction can not be simply neglected as dictated by Sutherland's theorem and as is widely believed by physicists.

We next do the same for the slope parameter and obtain

$$\alpha_\gamma = 0.0029. \quad (37)$$

This value is to be compared with the one-loop strong and electromagnetic corrections,

$$\alpha_{\text{str}} = 0.0179, \quad \alpha_{\text{em}} = -0.0011, \quad (38)$$

respectively. The first conclusion we draw comes from the comparison between the electromagnetic corrections at one- and two-loop levels. Although the two-loop calculation does not include the contribution of all the diagrams, it is three times bigger than the one-loop calculation. Since the one-loop correction is originating from the mass difference between charged and neutral pions, and the two-loop correction considered here is due to photon exchange between charged intermediate pions, we can claim that the effect of soft virtual photons is more important than the one for hard virtual photons. Obviously, this conclusion does no more hold if some cancellations occur between the various contributions if we consider all diagrams with a photon exchange. This point should constitute a sufficient motivation for calculating the full amplitude including the exchange of one photon between charged intermediate light mesons.

We draw the second conclusion by comparing the size of the electromagnetic correction to the size of the one-loop pure strong correction. If we consider only the contribution of the photon exchange diagram, we find that the former amounts to 17% of the latter. Adding both one- and two-loop corrections, we find that the electromagnetic interaction contributes about 11% of the pure strong interaction at one loop. If we add both strong and electromagnetic corrections, at the one-loop order for the first, and the two-loop order for the second, we obtain for the slope parameter

$$\alpha = \underbrace{0.0179}_{\mathcal{O}(p^4)} + \underbrace{-0.0011}_{\mathcal{O}(e^2 p^2)} + \underbrace{0.0029}_{\mathcal{O}(e^2 p^4)} = +0.0198, \quad (39)$$

to be compared with the most recent experimental value [7],

$$\alpha_{\text{exp}} = -0.0301 \pm 0.0035^{+0.0022}_{-0.0035}, \quad (40)$$

noting this endless sign discrepancy between the chiral perturbation theory prediction and observation.

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